

ADVANCED CAPITAL BUDGETING DECISIONS

CLASS 10

Topic Risk Analysis - Statistical Techniques

NPV Distribution given

Question 1:

Shivam Ltd. is considering two mutually exclusive projects A and B. Project A costs ₹ 36,000 and project B ₹ 30,000. You have been given below the net present value probability distribution for each project.

Project A		Project B	
NPV estimates (₹)	Probability	NPV estimates (₹)	Probability
15,000	0.2	15,000	0.1
12,000	0.3	12,000	0.4
6,000	0.3	6,000	0.4
3,000	0.2	3,000	0.1

- Compute the expected net present values of projects A and B.
- Compute the risk attached to each project i.e. standard deviation of each probability distribution.
- Compute the profitability index of each project.
- Which project do you recommend? State with reasons.

(Source: ICAI)

ANSWER:

i. Statement showing computation of expected net present value of Projects A and B:

Project A			Project B		
NPV Estimate (₹)	Probability	Expected Value	NPV Estimate	Probability	Expected Value
15,000	0.2	3,000	15,000	0.1	1,500
12,000	0.3	3,600	12,000	0.4	4,800
6,000	0.3	1,800	6,000	0.4	2,400
3,000	0.2	600	3,000	0.1	300
	1.0	EV = 9,000		1.0	EV = 9,000

ii. Computation of Standard deviation of each project

Project A

P	X	(X – EV)	P (X - EV) ²
0.2	15,000	6,000	72,00,000
0.3	12,000	3,000	27,00,000
0.3	6,000	- 3,000	27,00,000
0.2	3,000	- 6,000	72,00,000
			Variance = 1,98,00,000

Standard Deviation of Project A = $\sqrt{1,98,00,000} = ₹ 4,450$

Project B

P	X	(X – EV)	P (X - EV) ²
0.1	15,000	6,000	36,00,000
0.4	12,000	3,000	36,00,000
0.4	6,000	- 3,000	36,00,000
0.1	3,000	- 6,000	36,00,000
			Variance = 1,44,00,000

Standard Deviation of Project A = $\sqrt{1,44,00,000} = ₹ 3,795$

iii. Computation of profitability of each project

Profitability index = Discount cash inflow / Initial outlay

$$\text{In case of Project A : PI} = \frac{9,000 + 36,000}{36,000} = \frac{45,000}{36,000} = 1.25$$

$$\text{In case of Project B : PI} = \frac{9,000 + 30,000}{30,000} = \frac{39,000}{30,000} = 1.30$$

Measurement of risk is made by the possible variation of outcomes around the expected value and the decision will be taken in view of the variation in the expected value where two projects have

the same expected value, the decision will be the project which has smaller variation in expected value. In the selection of one of the two projects A and B, Project B is preferable because the possible profit which may occur is subject to less variation (or dispersion). Much higher risk is lying with project A.

CF Distribution given, Expected NPV coming same and hence decision is taken based on S.D of CF

Question 2:

Cyber Company is considering two mutually exclusive projects. Investment outlay of both the projects is ₹ 5,00,000 and each is expected to have a life of 5 years. Under three possible situations their annual cash flows and probabilities are as under:

Situation	Probabilities	Cash Flow (₹)	
		Project A	Project B
Good	0.3	6,00,000	5,00,000
Normal	0.4	4,00,000	4,00,000
Worse	0.3	2,00,000	3,00,000

The cost of capital is 7 per cent, which project should be accepted? Explain with workings.

(Source: ICAI)

ANSWER:

Project A

Expected Net Cash flow (ENCF)

$$0.3 (6,00,000) + 0.4 (4,00,000) + 0.3 (2,00,000) = 4,00,000$$

$$\sigma^2 = 0.3 (6,00,000 - 4,00,000)^2 + 0.4 (4,00,000 - 4,00,000)^2 + 0.3 (2,00,000 - 4,00,000)^2$$

$$\sigma = \sqrt{24,00,00,00,000}$$

$$\sigma = 1,54,919.33$$

$$\text{Present Value of Expected Cash Inflows} = 4,00,000 \times 4.100 = 16,40,000$$

$$\text{NPV} = 16,40,000 - 5,00,000 = 11,40,000$$

Project B

$$\text{ENCF} = 0.3 (5,00,000) + 0.4 (4,00,000) + 0.3 (3,00,000) = 4,00,000$$

$$\sigma^2 = 0.3 (5,00,000 - 4,00,000)^2 + 0.4 (4,00,000 - 4,00,000)^2 + 0.3 (3,00,000 - 4,00,000)^2$$

$$\sigma = \sqrt{6,00,00,00,000}$$

$$\sigma = 77,459.66$$

$$\text{Present Value of Expected Cash Inflows} = 4,00,000 \times 4.100 = 16,40,000$$

$$\text{NPV} = 16,40,000 - 5,00,000 = 11,40,000$$

Recommendation: NPV in both projects being the same, the project should be decided on the basis of standard deviation and hence project 'B' should be accepted having lower standard deviation, means less risky.

Hillier's model

Question 3:

Skylark Airways is planning to acquire a light commercial aircraft for flying class clients at an investment of ₹ 50,00,000. The expected cash flow after tax for the next three years is as follows:

₹

Year 1		Year 2		Year 3	
CFAT	Probability	CFAT	Probability	CFAT	Probability
14,00,000	0.1	15,00,000	0.1	18,00,000	0.2
18,00,000	0.2	20,00,000	0.3	25,00,000	0.5
25,00,000	0.4	32,00,000	0.4	35,00,000	0.2
40,00,000	0.3	45,00,000	0.2	48,00,000	0.1

The Company wishes to take into consideration all possible risk factors relating to airline operations. The company wants to know:

- The expected NPV of this venture assuming independent probability distribution with 6 per cent risk free rate of interest.
- The possible deviation in the expected value.
- How would standard deviation of the present value distribution help in Capital Budgeting decisions?

(Source: ICAI)

ANSWER:

i. Expected NPV

Year I			Year II			Year III		
CFAT	P	CF×P	CFAT	P	CF×P	CFAT	P	CF×P
14	0.1	1.4	15	0.1	1.5	18	0.2	3.6
18	0.2	3.6	20	0.3	6.0	25	0.5	12.5
25	0.4	10.0	32	0.4	12.8	35	0.2	7.0
40	0.3	12.0	45	0.2	9	48	0.1	4.8
	$\bar{X}$ or $\overline{CF}$	27.0		$\bar{X}$ or $\overline{CF}$	29.3			$\bar{X}$ or $\overline{CF}$ 27.9

NPV	PV factor @ 6%	Total PV
27	0.943	25.461
29.3	0.890	26.077
27.9	0.840	23.436
	PV of cash inflow	74.974
	Less: Cash outflow	50.000
	NPV	24.974

ii. Possible deviation in the expected value

Year I				
$X - \bar{X}$	$X - \bar{X}$	$(X - \bar{X})^2$	P1	$(X - \bar{X})^2 \times P1$
14 - 27	-13	169	0.1	16.9
18 - 27	-9	81	0.2	16.2
25 - 27	-2	4	0.4	1.6
40 - 27	13	169	0.3	50.7
				85.4

$$\sigma_1 = \sqrt{85.4} = 9.241$$

Year II				
$X - \bar{X}$	$X - \bar{X}$	$(X - \bar{X})^2$	P2	$(X - \bar{X})^2 \times P2$
15 - 29.3	-14.3	204.49	0.1	20.449
20 - 29.3	-9.3	86.49	0.3	25.947
32 - 29.3	2.7	7.29	0.4	2.916
45 - 29.3	15.7	246.49	0.2	49.298
				98.61

$$\sigma_2 = \sqrt{98.61} = 9.930$$

Year III				
$X - \bar{X}$	$X - \bar{X}$	$(X - \bar{X})^2$	P2	$(X - \bar{X})^2 \times P2$
18-27.9	-9.9	98.01	0.2	19.602
25-27.9	-2.9	8.41	0.5	4.205
35-27.9	7.1	50.41	0.2	10.082
48-27.9	20.1	404.01	0.1	40.401
				74.29

$$\sigma_3 = \sqrt{74.29} = 8.619$$

Standard deviation about the expected value:

$$\sigma = \sqrt{\frac{85.4}{(1.06)^2} + \frac{98.61}{(1.06)^4} + \frac{74.29}{(1.06)^6}} = 14.3696$$

- iii. Standard deviation is a statistical measure of dispersion; it measures the deviation from a central number i.e. the mean.

In the context of capital budgeting decisions especially where we take up two or more projects giving somewhat similar mean cash flows, by calculating standard deviation in such cases, we can measure in each case the extent of variation. It can then be used to identify which of the projects is least risky in terms of variability of cash flows.

A project, which has a lower coefficient of variation will be preferred if sizes are heterogeneous.

Besides this, if we assume that probability distribution is approximately normal we are able to calculate the probability of a capital budgeting project generating a net present value less than or more than a specified amount.

Hillier's model

Question 4:

Project X and Project Y are under the evaluation of XY Co. The estimated cash flows and their probabilities are as below:

Project X : Investment (year 0) ₹ 70 lakhs

Probability weights	0.30	0.40	0.30
Years	₹ lakhs	₹ lakhs	₹ lakhs
1	30	50	65
2	30	40	55
3	30	40	45

Project Y: Investment (year 0) ₹ 80 lakhs.

Probability weighted	Annual cash flows through life
	₹ lakhs
0.20	40
0.50	45
0.30	50

- Which project is better based on NPV, criterion with a discount rate of 10%?
- Compute the standard deviation of the present value distribution and analyse the inherent risk of the projects.

(Source: ICAI)

ANSWER:

a. Calculation of NPV of XY Co.:

	Project X	Cash flow	PVF	PV
Year 1	$(30 \times 0.3) + (50 \times 0.4) + (65 \times 0.3)$	48.5	0.909	44.09
Year 2	$(30 \times 0.3) + (40 \times 0.4) + (55 \times 0.3)$	41.5	0.826	34.28
Year 3	$(30 \times 0.3) + (40 \times 0.4) + (45 \times 0.3)$	38.5	0.751	28.91
				107.28
	NPV: $(107.28 - 70.00)$			(+) 37.28
	Project Y (For 1-3 Years)			
1-3	$(40 \times 0.2) + (45 \times 0.5) + (50 \times 0.3)$	45.5	2.487	113.16
	NPV $(113.16 - 80.00)$			(+) 33.16

b. Calculation of Standard deviation σ

As per Hiller's model

$$M = \sum_{i=0}^n (1+r)^{-i} M_i$$

$$\sigma^2 = \sum_{i=0}^n (1+r)^{-2i} \sigma_i^2$$

Hence

Project X

Year

$$1 \quad \sqrt{(30 - 48.5)^2 0.30 + (50 - 48.5)^2 0.40 + (65 - 48.5)^2 0.30} = \sqrt{185.25} = 13.61$$

$$2 \quad \sqrt{(30 - 41.5)^2 0.30 + (40 - 41.5)^2 0.40 + (55 - 41.5)^2 0.30} = \sqrt{95.25} = 9.76$$

$$3 \quad \sqrt{(30 - 38.5)^2 0.30 + (40 - 38.5)^2 0.40 + (45 - 38.5)^2 0.30} = \sqrt{35.25} = 5.94$$

Standard Deviation about the expected value

$$= \sqrt{\frac{185.25}{(1+0.10)^2} + \frac{95.25}{(1+0.10)^4} + \frac{35.25}{(1+0.10)^6}}$$

$$= \sqrt{\frac{185.25}{1.21} + \frac{95.25}{1.4641} + \frac{35.25}{1.7716}} = \sqrt{153.10 + 65.06 + 19.90}$$

$$= \sqrt{238.06} = 15.43$$

Project Y (For 1-3 Years)

$$\sqrt{(40 - 45.5)^2 0.20 + (45 - 45.5)^2 0.50 + (50 - 45.5)^2 0.30} = \sqrt{12.25} = 3.50$$

Standard Deviation about the expected value

$$\begin{aligned}
 &= \sqrt{\frac{12.25}{(1+0.10)^2} + \frac{12.25}{(1+0.10)^4} + \frac{12.25}{(1+0.10)^6}} \\
 &= \sqrt{\frac{12.25}{1.21} + \frac{12.25}{1.4641} + \frac{12.25}{1.7716}} = \sqrt{10.12+8.37+6.91} \\
 &= \sqrt{25.4} = 5.03
 \end{aligned}$$

Analysis: Project Y is less risky as its Standard Deviation is less than Project X.

Probability Tree

Question 5:

Following are the estimates of the net cash flows and probability of a new project of M/s X Ltd.:

	Year	P = 0.3	P = 0.5	P = 0.2
Initial investment	0	4,00,000	4,00,000	4,00,000
Estimated net after tax cash inflows per year	1 to 5	1,00,000	1,10,000	1,20,000
Estimated salvage value (after tax)	5	20,000	50,000	60,000

Required rate of return from the project is 10%. Find:

- The expected NPV of the project.
- The best case and the worst case NPVs.
- The probability of occurrence of the worst case if the cash flows are perfectly dependent overtime and independent overtime.
- Standard deviation and coefficient of variation assuming that there are only three streams of cash flow, which are represented by each column of the table with the given probabilities.
- Coefficient of variation of X Ltd. on its average project which is in the range of 0.95 to 1.0. If the coefficient of variation of the project is found to be less risky than average, 100 basis points are deducted from the Company's cost of Capital

Should the project be accepted by X Ltd?

(Source: ICAI)

ANSWER:

i. Expected cash flows:-

Year		Net cash flows	P.V.	PV. @ 10%
0	(4,00,000 x 1)	(-)4,00,000	1.000	(-)4,00,000
1 to 4	(1,00,000x0.3+1,10,000x0.5 + 1,20,000 x 0.2)	1,09,000	3.170	3,45,530
5	[1,09,000 + (20,000 x 0.3 + 50,000 x 0.5 + 60,000 x 0.2)]	1,52,000	0.621	94,392
		NPV=		39,922

ii. ENPV of the worst case

$1,00,000 \times 3.790 = ₹ 3,79,000$ (Students may have 3.791 also the values will change accordingly)

$20,000 \times 0.621 = ₹ 12,420/-$

$ENPV = (-) 4,00,000 + 3,79,000 + 12,420 = (-) ₹ 8,580/-$

ENPV of the best case

$ENPV = (-) 4,00,000 + 1,20,000 \times 3.790 + 60,000 \times 0.621 = ₹ 92,060/-$

iii. a. Required probability = 0.3

b. Required probability = $(0.3)^5 = 0.00243$

iv. The base case NPV = $(-) 4,00,000 + (1,10,000 \times 3.79) + (50,000 \times 0.621) = ₹ 47,950/-$

$ENPV = 0.30 \times (-) 8580 + 0.5 \times 47950 + 92060 \times 0.20 = ₹ 39,813/-$

Therefore,

$$\sigma_{ENPV} = \sqrt{0.3(-8580 - 39,813)^2 + 0.5(47950 - 39,813)^2 + 0.2(92,060 - 39,813)^2} = ₹ 35,800/-$$

Therefore, $CV = 35,800/39,813 = 0.90$

v. Risk adjusted out of cost of capital of X Ltd. = $10\% - 1\% = 9\%$.

NPV

Year	Expected net cash flow	PV @ 9%	
0	(-) 4,00,000	1.000	(-) 4,00,000
1 to 4	1,09,000	3.240	3,53,160
5	1,52,000	0.650	98,800
		ENPV =	51,960

Therefore, the project should be accepted.